MI-0196

Translating Measured Multipoles onto the Reference Orbit in Recycler Combined Function Magnets

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The Recycler is to be constructed utilizing combined function magnets. It is anticipated that production measurements of these magnets will utilize a straight rotating coil aligned with the transverse center of the magnet. Since the reference orbit in these magnets follows an arc with ~11 mm sagitta the beam will experience magnetic fields that are different than those measured with the probe. The purpose of this note is to provide a prescription for translating multipoles as measured by a straight probe into multipole content as seen on the reference orbit and to evaluate the significance of these effects.

Multipole Translation

The field within any magnet can be described through a multipole expansion of the form

$$B_y + iB_x = B_0 \sum_n (b_n + ia_n)(x+iy)^n \tag{1}$$

where a Cartesian coordinate is referenced with z along the longitudinal axis of the magnet, y in the vertical direction, and x in the horizontal plane perpendicular to z. B_x and B_y refer to the x and y field components averaged along the probe axis over the length of the magnet. The b_n (a_n) are referred to as the normal (skew) multipole components and at Fermilab are measured in "units" with one unit corresponding to a multipole strength of 10^{-4} of the nominal dipole field strength (B_0) as observed 1" off axis.

Suppose we wish to refer the multipoles to a position displaced from probe axis by an amount x_R. The resulting multipoles can be calculated by replacing x with x-x_R in equation (1). Some straight-forward algebra yields:

$$\begin{cases} \tilde{b}_n = \sum\limits_{m \geq 0} \dfrac{(m+n)!}{m!\,n!} b_{m+n} x_R^m \\ \tilde{a}_n = \sum\limits_{m \geq 0} \dfrac{(m+n)!}{m!\,n!} a_{m+n} x_R^m \end{cases} \tag{2}$$

where $\tilde{b}_n$ and $\tilde{a}_n$ are the multipoles as seen from the displaced position x_R.

Multipoles Averaged over the Reference Orbit

The reference orbit in the Recycler combined function magnets deviates significantly from a straight line. The actual trajectories are described in MI-0195. Multipoles as seen by the beam are calculated by applying equation (2) piece wise, in 20 cm steps, to the reference trajectory over the length of the magnet. Results are given in Table 1. Multipoles as viewed along the measured (straight, along the magnet transverse center) and as viewed along the reference orbit are listed for each of the four gradient magnet types. The values is the "Measured" column are the nominal values for Recycler lattice RRV10 as used in the tracking simulations. The values in the "Reference Orbit" column are calculated assuming a magnet alignment that provides a reference orbit entry angle of one half the total bend and an entry position displaced radially inward from the center by one half the value of the sagitta.

Table 1: Multipoles, measured in Fermilab units, as seen on the Recycler reference orbit, and as measured using a straight probe placed in the transverse center of the four variety of Recycler magnets.

	Long Focusing		Long Defocusing		Short Focusing		Short Defocusing	
	Measured	Reference Orbit	Measured	Reference Orbit	Measured	Reference Orbit	Measured	Reference Orbit
b0	0	36.75	0	-37.37	0	31.36	0	-34.35
b1	616.16	617.22	-590.04	-591.88	1346.31	1346.33	-1380.93	-1380.91
b2	8.60	8.72	-14.98	-14.85	0.20	0.24	0.20	0.24
b3	0.50	0.58	0.50	0.58	0.50	0.52	0.50	0.52
b4	0.20	0.27	0.20	0.27	0.20	0.22	0.20	0.22
b5	0.10	0.14	0.10	0.14	0.10	0.11	0.10	0.11
b6	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
a1	0.20	0.16	0.20	0.15	0.20	0.18	0.20	0.18
a2	-0.50	-0.48	-0.50	-0.48	-0.50	-0.49	-0.50	-0.49
a3	0.25	0.18	0.25	0.18	0.25	0.22	0.25	0.22
a4	-0.20	0.01	-0.20	0.00	-0.20	-0.17	-0.20	-0.17
a5	-0.25	0.03	-0.25	0.05	-0.25	-0.14	-0.25	-0.13
a6	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80

The table shows that the amount of excess (deficit) bending due to the gradient component of the focusing (defocusing) combined function magnets is significant--approximately 37 units. If uncorrected this would lead to a closed orbit distortion of several tens of millimeters. Likewise, the sextupole component of the long combined function magnets produces an over focusing of approximately 1.5 units. If uncorrected this effect would cause a tune shift of about .04. Both the closed orbit distortion and the tune shift are beyond the range of what would be regarded as acceptable, and so these effects must be taken into account in the design (or alignment) of the magnets.

Feed down effects are also evident in the higher order multipoles, although they are not terribly significant.. It is suggested that these be dealt with simply by correctly incorporating into the tracking simulations.

Conclusions

The reference orbit in the Recycler combined function magnets follows a trajectory that deviates significantly from a straight line. In evaluating potential effects two assumptions are made:

1. Magnet strength and multipole content will be measured with a straight probe positioned at the nominal (transverse) magnet center.

2. Magnets will be aligned in the ring such that the reference orbit entry angle is one half the nominal magnet bend angle, and the reference orbit entry position is displaced radially inward from the nominal (transverse) magnet center by an amount corresponding to one half the orbit sagitta in the magnet.

The most significant effects are an over (under) bending in the focusing (defocusing) magnets and an over focusing in the long magnets is affected. These effects, if uncompensated, would lead to unacceptable orbit distortions and tune shifts.

Two potential means of compensation can be contemplated. The first is to install the magnets with a transverse offset to compensate for the feed down effects. The required offset is 1.6 mm (radially outward relative to assumption 2) in the long magnets (both F and D) and 0.6 mm (radially outward relative to assumption 2) in the short magnets (both F and D). Note: <u>The reference orbit of the Recycler does not move in this situation</u>--only the position of the reference orbit relative to the magnets. Such an alignment strategy will reduce systematic shifts in bending and focusing strength to less than 0.1 unit. The second possibility for compensation is to construct focusing (defocusing) magnets that are weaker (stronger) than the nominal specification as measured with a straight probe through the center. The required adjustments are given in Table 2.

If mechanically acceptable, the first option is probably preferred because it reduces the potential for confusion in the measuring process and also tends to minimize feed down effects in the higher order multipoles. If this option is not feasible mechanically, then the second option should be adopted.

Feed down effects also impact the higher order multipoles. While these are not terribly significant, it is suggested that they be correctly incorporated into the tracking simulations.

Table 2: Required strength adjustment of the integrated bending and focusing components, as measured with a straight probe, to provide nominal performance on the reference orbit subject to assumptions 1) and 2) defined in the text.

Multipole	Long Focusing	Long Defocusing	Short Focusing	Short Defocusing
b0	-36.8	+37.4	-31.4	+34.4
b1	-1.1	+1.8	0.0	0.0